Surface electromagnetic waves at an anisotropically conducting artificial interface

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Surface electromagnetic waves propagating along an anisotropically conducting interface of two different dielectrics have been theoretically investigated. The flat surface of the interface contains a one-dimensional array of thin metal wires. It was assumed that both the lattice constant of the array and the diameter of the wires are far less than the lengths of the surface waves. It has been shown that the surface electromagnetic waves may propagate along the interface at frequencies which are far lower than the plasma frequency of a metal, and the electric field of the waves is always perpendicular to the wires in any propagation directions. The existence conditions, dispersion relations, and energy fluxes of the surface waves have been derived. It has been demonstrated that the surface electromagnetic waves can be excited by means of the transition radiation and beam instability effects.

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I. INTRODUCTION

Nowadays a good deal of attention is focused on studies into the properties of surface electromagnetic waves (SEMWs) because they are extensively used in modern electronic devices. The SEMWs with TM polarization referred to as the surface plasmon polaritons (SPPs) are used for up-to-date applications, such as plasmon waveguides, aperture arrays for enhanced light transmission, and various geometries for surface-enhanced sensing.[1] The concept of a waveguide for SPPs is based upon the insulator/metal/insulator structure that consists of a thin metal stripe (on the order of 10 nm) sandwiched between two thick dielectrics. If the strip is embedded in a homogeneous dielectric host, the multilayer system sustains the long-ranging SPP mode that can propagate for distances over multiple millimeters in the near infrared (see Ref. 2). The long-range SPP propagation along subwavelength nanowires has been investigated in Ref. 3. We should like to highlight the so-called designer SPPs on corrugated surfaces. In Refs. 4 and 5 it has been shown that the SPP-like bound electromagnetic surface waves (the designer SPPs) at tetrahertz (THz) frequencies can be sustained by a perfect conductor with the periodically corrugated surface. The dispersion relation of such surface waves can be engineered via the geometry of the surface. The effects of propagation and focusing of THz SPPs on periodically corrugated metal wires have been found in Ref. 6. The well-bounded SEMWs guided by metallic wedges (the so-called wedge plasmon polaritons) at a telecom wavelength have been theoretically studied in Ref. 7.

In the present paper we have looked into the possibility of well-bounded SEMWs propagation at the flat interface of two different nonabsorbing dielectrics in the case where the interface contains an array of perfectly conducting parallel thin wires (say, metal wires). The lattice constant of the array and the diameter of wires are supposed to be far less than the wavelength. The SEMWs under consideration can likewise be called as the designer SEMWs because these waves propagate along the artificial interface at frequencies which are far lower than the metal plasma frequency $\omega_p = \sqrt{4\pi e^2 N/m}$ (where e is the electron charge, N is the bulk electron density, and m is the electron effective mass).

II. DISPERSION EQUATION AND ENERGY FLUX

Let the flat interface of two nonabsorbing dielectric media be located in xz plane (see Fig. 1). The half space $y<0$ is described by dielectric constant ε_1 and the half space $y>0$ is described by dielectric constant ε_2. The infinite one-dimensional array of parallel thin metal wires is placed in the interface plane. The lattice constant d_1 of the array along with the diameter of wires d_2 are taken to be far less than the wavelength of the SEMW (i.e., $d_1, d_2 \ll \lambda$ where λ is the SEMW wavelength). Because the lattice constant of the array and the wires diameter is far less than the wavelength, the interface can be interpreted as a homogeneous surface with anisotropic conducting properties. This fact allows the

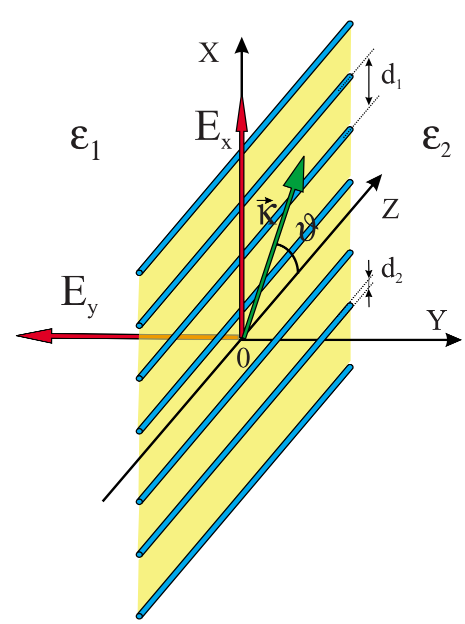

FIG. 1. (Color online) Geometry of the problem.

use of the homogeneous boundary condition at the interface.[8]

Now show that the SEMWs with components $(E_{x\ell}, E_{y\ell}, 0)$, $(H_{x\ell}, H_{y\ell}, H_{z\ell})$ (where $\ell = 1, 2$ is the number of the dielectric medium) can propagate at frequencies where the conductivity of metal wires is an infinitely large. This implies that the frequencies of SEMWs is far lower than ω_p and, therefore, the metal may be considered as a perfect conductor. For metals in which the bulk electron density N is of the order of $N \sim 10^{28}$ m^{-3} and the electron effective mass m is equal to the mass of a free electron m_0 we obtain $\omega_p \sim 5 \times 10^{15}$ s^{-1}. Hence, the condition $\omega \ll \omega_p$ can be fulfilled at THz frequencies or lower.

We specify the fields of SEMWs as

$$\vec{E}_\ell = \vec{E}_{0\ell} \exp[i(\vec{\kappa}\vec{\rho} + k_{y\ell} y - \omega t)], \quad (1)$$

$$\vec{H}_\ell = \vec{H}_{0\ell} \exp[i(\vec{\kappa}\vec{\rho} + k_{y\ell} y - \omega t)], \quad (2)$$

where $\vec{\kappa} = (k_x, k_z)$ is the wave vector in xz plane, $\vec{\rho} = (x, z)$ is the radius vector in xz plane

$$k_{y1} = -i\sqrt{\kappa^2 - \frac{\omega^2}{c^2}\varepsilon_1}, \quad k_{y2} = i\sqrt{\kappa^2 - \frac{\omega^2}{c^2}\varepsilon_2}, \quad (3)$$

where c is the speed of light in vacuum. The signs in the right-hand sides of Eq. (3) are due to the confinement of SEMWs to the interface. Consider the Maxwell equations for electromagnetic fields in dielectric media

$$\nabla \times \vec{H}_\ell = \frac{\varepsilon_\ell}{c}\frac{\partial \vec{E}_\ell}{\partial t}, \quad \nabla \times \vec{E}_\ell = -\frac{1}{c}\frac{\partial \vec{H}_\ell}{\partial t}, \quad (4)$$

$$\mathrm{div}\, \vec{E}_\ell = 0, \quad \mathrm{div}\, \vec{H}_\ell = 0. \quad (5)$$

From Eqs. (4) and (5) we obtain the following expressions for the components of electric and magnetic fields:

$$E_{y\ell} = -\frac{k_x}{k_{y\ell}} E_{x\ell}, \quad H_{x\ell} = \frac{c k_x k_z}{\omega k_{y\ell}} E_{x\ell}. \quad (6)$$

$$H_{y\ell} = \frac{c k_z}{\omega} E_{x\ell}, \quad H_{z\ell} = -\frac{c(k_x^2 + k_{y\ell}^2)}{\omega k_{y\ell}} E_{x\ell}. \quad (7)$$

In order to derive the dispersion relation for SEMWs it is necessary to satisfy a certain continuity conditions at $y=0$, namely, the continuity conditions for the tangential components of the electric $E_{x\ell}$ and magnetic $H_{z\ell}$ fields

$$E_{x1}(0) = E_{x2}(0), \quad H_{z1}(0) = H_{z2}(0). \quad (8)$$

It should be noted that both the normal component of the electric displacement $D_{y\ell} = \varepsilon_l E_{y\ell}$ and tangential component of magnetic field $H_{x\ell}$ suffer discontinuities caused by excitation of surface current in the wires. The discontinuity of $D_{y\ell}$ is equal to the surface charge density n^{2D} and the discontinuity of $H_{x\ell}$ is equal to the surface current density j_z^{2D}

$$\varepsilon_2 E_{y2}(0) - \varepsilon_1 E_{y1}(0) = 4\pi e n^{2D}, \quad (9)$$

$$H_{x2}(0) - H_{x1}(0) = -\frac{4\pi}{c} j_z^{2D}, \quad (10)$$

where $n^{2D} \sim \exp[i(k_z z - \omega t)]$ and $j_z^{2D} \sim \exp[i(k_z z - \omega t)]$ satisfy the continuity equation

$$\frac{\partial e n^{2D}}{\partial t} + \frac{\partial j_z^{2D}}{\partial z} = 0. \quad (11)$$

From Eqs. (6), (7), and (9)–(11) we obtain the expressions connecting the field components $E_{x\ell}$ with the surface current density j_z^{2D}

$$E_{x1}(0) = \frac{4\pi k_{y1}}{\omega k_x k_z (\varepsilon_2 - \varepsilon_1)} \left(\frac{\omega^2}{c^2}\varepsilon_2 - k_z^2 \right) j_z^{2D}, \quad (12)$$

$$E_{x2}(0) = \frac{4\pi k_{y2}}{\omega k_x k_z (\varepsilon_2 - \varepsilon_1)} \left(\frac{\omega^2}{c^2}\varepsilon_1 - k_z^2 \right) j_z^{2D}. \quad (13)$$

Substituting Eqs. (6) and (7) into Eq. (8), we find the dispersion relation for SEMWs

$$\frac{1}{k_{y2}} \left(\frac{\omega^2}{c^2}\varepsilon_2 - k_z^2 \right) = \frac{1}{k_{y1}} \left(\frac{\omega^2}{c^2}\varepsilon_1 - k_z^2 \right). \quad (14)$$

Using the definitions of k_{y1} and k_{y2} from Eq. (3), the dispersion relation, Eq. (14), can be rewritten as the quadratic equation of ω^2 with the following roots:

$$\omega_{(\pm)}^2 = \frac{c^2 k_z^2}{2\varepsilon_1\varepsilon_2 \cos^2\vartheta} [\varepsilon_2 + \varepsilon_1 \pm \sqrt{(\varepsilon_2 - \varepsilon_1)^2 + 4\varepsilon_1\varepsilon_2 \sin^4\vartheta}], \quad (15)$$

where ϑ is the angle between vector $\vec{\kappa}$ and the positive direction of axis z (see Fig. 1), $\tan^2\vartheta = k_x^2/k_z^2$. Note that only one of two roots in Eq. (15) has a physical meaning and corresponds to sought SEMWs. In order to choose the appropriate root, we have to formulate the additional conditions of SEMWs existence. For the sake of definiteness, we suppose that

$$\varepsilon_1 < \varepsilon_2.$$

Then the existence conditions can be written as

$$\frac{\omega^2}{c^2}\varepsilon_2 < \kappa^2, \quad \frac{\omega^2}{c^2}\varepsilon_1 < k_z^2 < \frac{\omega^2}{c^2}\varepsilon_2. \quad (16)$$

Substituting Eq. (15) into Eq. (16) and performing necessary calculations, we find that the existence conditions are fulfilled at $\omega^2 = \omega_{(-)}^2$ for $0 < \vartheta < \pi/2$. Henceforward, we will omit index "−" at ω^2.

For $\vartheta \to \pi/2$ (i.e., for $k_x^2 \gg k_z^2$) the solution for ω^2 takes the form

$$\omega^2 \approx \frac{2c^2 k_z^2}{\varepsilon_1 + \varepsilon_2}. \quad (17)$$

It is worthwhile to emphasize that the SEMWs described by dispersion relation in Eq. (17) closely resemble the so-called surface helicons.[9] By making the analogy with the system investigated in Ref. 9, one can see that in the case under consideration the wires made of a perfect conductor act as

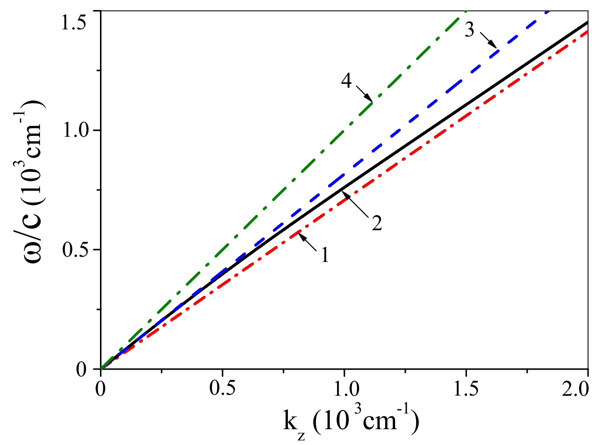

FIG. 2. (Color online) Dispersion curves of the SEMWs at the interface between two different dielectrics for $\varepsilon_1=1$ and $\varepsilon_2=2$. Curves 1 corresponds to the light line of dielectric $2(k_x=0)$, curve 2 is for $k_x=5\times10^4$ m^{-1}, curve 3 is for $k_x^2\gg k_z^2$, and curve 4 is the light line of the dielectric 1.

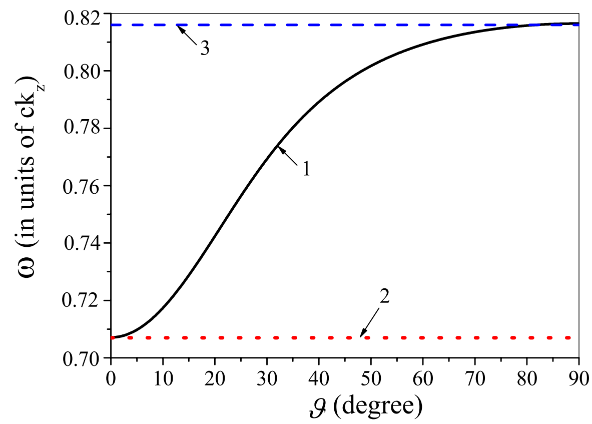

FIG. 3. (Color online) The angular dependence (curve 1) of the normalized frequency of SEMWs for $\varepsilon_1=1$ and $\varepsilon_2=2$. Curve 2 corresponds to $\omega/(ck_z)=1/\sqrt{\varepsilon_2}$ and curve 3 is for $\omega/(ck_z)=\sqrt{2/(\varepsilon_1+\varepsilon_2)}$.

the magnetic field in Ref. 9. In addition, the waves described by dispersion relation in Eq. (17) are the delayed ones, i.e., their phase velocities v_{ph} are far lower than the speed of light in vacuum

$$v_{ph}\approx c\sqrt{\frac{2}{\varepsilon_1+\varepsilon_2}\left(\frac{\pi}{2}-\vartheta\right)^2}, \tag{18}$$

where $0<\pi/2-\vartheta\ll 1$. Hereafter, we will demonstrate that this particular property allows an effective coupling of the delayed waves and an electron-beam wave.

Figure 2 shows the k_z dependence of ω/c for a number of k_x values at $\varepsilon_1=1$ and $\varepsilon_2=2$. In Fig. 2 curve 1 corresponds to the light line of dielectric $2(k_x=0)$, curve 2 is for $k_x=5\times10^4$ m^{-1}, curve 3 is for $k_x^2\gg k_z^2$ (i.e., $\vartheta\to\pi/2$), and curve 4 is the light line of dielectric 1. The dispersion curves that correspond to $0<k_x<\infty$ lie in the range between curve 1 and curve 3. Let us examine the confinement of the SEMWs by taking a closer look at their dispersion curves.

As seen from Fig. 2, for $k_z\to0$ the dispersion curve 2 asymptotically approaches curve 3. It means that the SEMWs propagate almost perpendicular to the wires. In this case, specifically for $k_z\to0$ and $k_x\neq0$, the frequency ω is approximately described by Eq. (17) and the localization depth $\delta_\ell=2\pi/|k_{y\ell}|$ goes to the wavelength $\lambda=2\pi/\kappa$

$$\frac{\delta_\ell}{\lambda}=\frac{\kappa}{\sqrt{\kappa^2-\varepsilon_\ell\omega^2/c^2}}\to1. \tag{19}$$

Note that the localization depth δ_ℓ defines the evanescent decay length of the field perpendicular to the interface, which quantifies the confinement of the wave. From Eqs. (15) and (19) it follows that for $k_z^2\sim k_x^2$ the decay length is $\delta_\ell>\lambda$ whereas for $k_z^2\gg k_x^2$ we get $\delta_1\to\lambda/\sqrt{1-\varepsilon_1/\varepsilon_2}$ and $\delta_2\to\infty$. Indeed, for $k_z^2\gg k_x^2$ the dispersion curve 2 asymptotically approaches curve 1 and the waves extend over many wavelength into dielectric 2. Therefore, the SEMWs are well bounded in the both dielectric media, chiefly for $k_x^2\gg k_z^2$ with $\delta_\ell\approx\lambda$.

The ϑ dependence (curve 1) of $\omega/(ck_z)$ for $\varepsilon_1=1$, $\varepsilon_2=2$ is shown in Fig. 3. In Fig. 3 curve 2 corresponds to $\omega/(ck_z)=1/\sqrt{\varepsilon_2}$ and curve 3 is for $\omega/(ck_z)=\sqrt{2/(\varepsilon_1+\varepsilon_2)}$. As evident from Fig. 3, the highest frequency can be achieved only by SEMWs propagating almost perpendicular to the wires.

Note that for $\varepsilon_1=\varepsilon_2$ the z components of the SEMWs magnetic field tend to zero on both sides of the interface and the dispersion relation of the SEMWs becomes $\omega=ck_z/\sqrt{\varepsilon}$. These SEMWs may be thought of as a degenerate case of the SEMWs considered above. The degeneracy occurs on the k_x component of the wave vector and it means that the SEMWs with dispersion relation $\omega=ck_z/\sqrt{\varepsilon}$ at any specified frequency propagate over the entire angle interval $0<\vartheta<\pi/2$ with phase velocities $0<v_{ph}<c/\sqrt{\varepsilon}$. The possibility of existence of the SEMWs with dispersion relation $\omega=ck_z/\sqrt{\varepsilon}$ was studied in Ref. 10. The excitation of the SEMWs with dispersion relation $\omega=ck_z$ by an electric dipole was investigated in Ref. 11.

Using the well-known expression for the time-averaged Poynting vector, the energy flux of SEMWs in each of the dielectrics can be expressed as

$$\langle\vec{S}_\ell\rangle=\frac{c}{8\pi}\mathrm{Re}[\vec{E}_\ell,\vec{H}_\ell^*], \tag{20}$$

where the angle brackets signify the averaging operation over the field oscillation period. From Eqs. (6), (7), and (20) we obtain the following expressions for $\langle S_{x\ell}\rangle,\langle S_{z\ell}\rangle$:

$$\langle S_{x\ell}\rangle=\frac{c^2k_x}{8\pi\omega|k_{y\ell}|^2}\left(\frac{\omega^2}{c^2}\varepsilon_\ell-k_z^2\right)|E_x(0)|^2\exp[-2y\ \mathrm{Im}(k_{y\ell})], \tag{21}$$

$$\langle S_{z\ell}\rangle=\frac{c^2k_z}{8\pi\omega|k_{y\ell}|^2}(k_x^2+|k_{y\ell}|^2)|E_x(0)|^2\exp[-2y\ \mathrm{Im}(k_{y\ell})]. \tag{22}$$

Note that $\langle S_{y\ell}\rangle=0$ for the nonabsorbing dielectrics. As indicated from Eqs. (21) and (22), $\langle S_{x1}\rangle<0$, $\langle S_{x2}\rangle>0$, and

$\langle S_{z1}\rangle, \langle S_{z2}\rangle > 0$ in accordance with conditions (16). It can easily be shown that in the general case the tangential component of the Poynting vector $\langle \vec{S}_{\tau\ell}\rangle = (\langle S_{x\ell}\rangle, \langle S_{z\ell}\rangle)$ forms an acute angle with wave vector $\vec{\kappa}$ in both adjacent media

$$\langle \vec{S}_{\tau\ell}\rangle \vec{\kappa} = \frac{c^2}{8\pi\omega|k_{y\ell}|^2}\left(k_x^2\frac{\omega^2}{c^2}\varepsilon_\ell + k_z^2|k_{y\ell}|^2\right)|E_x(0)|^2 \times \exp[-2y\,\mathrm{Im}(k_{y\ell})] \geq 0. \quad (23)$$

It qualitatively differs, for instance, from the mutual orientation of vectors $\langle \vec{S}_\tau\rangle$ and $\vec{\kappa}$ for SEMWs propagating at the interface between a plasmalike medium and a dielectric. Indeed, in a plasmalike medium $\langle \vec{S}_\tau\rangle\vec{\kappa} < 0$ and $\langle \vec{S}_\tau\rangle\vec{\kappa} > 0$ in a dielectric. As seen from Eq. (23), the product of $\langle \vec{S}_\tau\rangle\vec{\kappa}$ is equal to zero when $\vartheta \to \pi/2$. In this case the main part of the energy flux propagates along the wires and the energy flux components $\langle S_{z\ell}\rangle$ for $\ell = 1, 2$ become equal to each other

$$\langle S_{x\ell}\rangle \to 0, \quad \langle S_{z\ell}\rangle \to \frac{\pi}{c}\sqrt{\frac{2}{\varepsilon_1+\varepsilon_2}}|j_z^{2D}|^2 \times \exp[-2y\,\mathrm{Im}(k_{y\ell})], \quad (24)$$

where $k_{y1} = -|k_x|$ and $k_{y2} = |k_x|$. The equal values of $\langle S_{z\ell}\rangle$ results from the equal decay lengths δ_ℓ of SEMWs in the adjacent media as $\vartheta \to \pi/2$.

In the conclusion of this section, note that if we take into account the finite conductivity of wires σ, then the additional SEMWs appear with the electric field polarized in the plane that contains the "y" axis and the "z" axis (see Fig. 1). As a result, the dispersion relation of the superposition of the SEMWs with $E_z = 0$ and the additional SEMWs takes the form

$$\frac{1}{k_{y2}}\left(\frac{\omega^2}{c^2}\varepsilon_2 - k_z^2\right) - \frac{1}{k_{y1}}\left(\frac{\omega^2}{c^2}\varepsilon_1 - k_z^2\right) = -\frac{\omega}{4\pi\sigma d_2}\frac{(k_{y2}-k_{y1})(\varepsilon_2 k_{y1} - \varepsilon_1 k_{y2})}{k_{y1}k_{y2}}. \quad (25)$$

From Eq. (25) it is seen that in the limit of $\sigma \to \infty$ the dispersion relation in Eq. (25) goes over into the dispersion relation in Eq. (14). Given the finite conductivity, leads to the attenuation of the SEMWs in the propagation direction. For instance, at $\vartheta \to \pi/2$ the expression (17) becomes

$$\omega^2 = \frac{2c^2k_z^2}{\varepsilon_1+\varepsilon_2} - i\frac{c^3k_x^2}{\pi\sigma d_2\sqrt{2(\varepsilon_1+\varepsilon_2)}}. \quad (26)$$

From Eq. (26) one can obtain the damping decrement Γ and the propagation length L of the SEMWs

$$\Gamma = |\mathrm{Im}\{\omega\}| = \frac{c^2|k_x|\cos\vartheta\tan^2\vartheta}{4\pi\sigma d_2}, \quad (27)$$

$$L = \frac{4\sqrt{2}\pi\sigma d_2}{\sqrt{\varepsilon_1+\varepsilon_2}c|k_x|\tan^2\vartheta} \quad (28)$$

at

$$1 \ll \tan^2\vartheta \ll \frac{2\pi\sigma d_2}{c}\sqrt{\frac{2}{\varepsilon_1+\varepsilon_2}}. \quad (29)$$

Let us make the numerical calculations of Γ and L at gigahertz (GHz) and THz frequencies for copper wires at $\omega_p \simeq 1.6\times10^{16}$ s^{-1}, $d_2 = 10^{-5}$ m, $\varepsilon_1 = 1$, $\varepsilon_2 = 2$, and $\vartheta = 85°$. For $k_x = 10^3$ m^{-1} at $\omega \simeq 20$ GHz we have $\Gamma \simeq 1.5\times10^7$ s^{-1} ($\Gamma/\omega_p \simeq 10^{-9}$) and $L \simeq 1.4$ m. For $k_x = 5\times10^4$ m^{-1} at $\omega \simeq 1$ THz we have $\Gamma \simeq 7.7\times10^8$ s^{-1} ($\Gamma/\omega_p \simeq 5\times10^{-8}$) and $L \simeq 2.8\times10^{-2}$ m. One should pay special attention to sufficiently large values L and small decay lengths δ_ℓ of the SEMWs. For example, at GHz frequencies we have $\delta_1 \simeq \delta_2 \simeq 0.6\times10^{-2}$ m and at THz frequencies we have $\delta_1 \simeq \delta_2 \simeq 10^{-4}$ m. For comparison we note that in Ref. 12 the THz SPPs at the air/gold interface are experimentally investigated. It has been found that the SPP propagation length is about 1.8×10^{-2} m and the SPP air decay length is larger than 1.6×10^{-2} m. Besides, it should be noted that the z component value of the electric field of the additional SEMW is small as compared to the x-component value of the electric field of the SEMW with $E_z = 0$. Indeed, for above-mentioned parameters of the wires and the dielectric media we find that $|E_z(0)|/|E_x(0)| < 10^{-4}$ at GHz and THz frequencies.

III. EXCITATION OF THE SEMWs BY THE TRANSITION-RADIATION EFFECT

Now examine the case where SEMWs considered above (at $\sigma \to \infty$) can be excited using the transition-radiation effect of an electron that moves along the normal to the interface. Let an electron move from dielectric 1 to dielectric 2 at a velocity $v_0 \ll c$. The electron-charge density Q is determined by the formula

$$Q(\vec{r},t) = e\,\delta(x)\,\delta(y - v_0 t)\,\delta(z), \quad (30)$$

where $\delta(x)$ is the Dirac delta function. The electromagnetic fields of the electron are expressed in terms of Fourier integrals

$$\vec{E}_\ell^e(\vec{r},t) = \int \vec{E}_\ell^e(\vec{k}^e,\omega)\exp[i(\vec{k}^e\vec{r} - \omega t)]d\vec{k}^e d\omega, \quad (31)$$

where $\vec{k}^e = (k_x, k_y^e, k_z)$. The Fourier components $\vec{E}_\ell^e(\vec{k}^e,\omega)$ and $\vec{H}_\ell^e(\vec{k}^e,\omega)$ for a single electron are[8]

$$\vec{E}_\ell^e(\vec{k}^e,\omega) = \frac{ei}{2\pi^2\varepsilon_\ell}\frac{\omega\varepsilon_\ell\vec{v}_0/c^2 - \vec{k}^e}{(k^e)^2 - \omega^2\varepsilon_\ell/c^2}\delta(k_y^e v_0 - \omega), \quad (32)$$

$$\vec{H}_\ell^e(\vec{k}^e,\omega) = \frac{c}{\omega}[\vec{k}^e, \vec{E}_\ell^e(\vec{k}^e,\omega)]. \quad (33)$$

We assume that the radiation field is the superposition of electromagnetic waves (EMWs) of E—and H types. The components of E-type EMWs are

$$(E_{x\ell}^{(E)}, E_{y\ell}^{(E)}, E_{z\ell}^{(E)}), \quad (H_{x\ell}^{(E)}, 0, H_{z\ell}^{(E)}). \quad (34)$$

The components of H-type EMWs are

$$(E_{x\ell}^{(H)},0,E_{z\ell}^{(H)}),\quad (H_{x\ell}^{(H)},E_{y\ell}^{(H)},H_{z\ell}^{(H)}). \tag{35}$$

The radiation fields of both types we expressed in terms of the following Fourier integrals:

$$\vec{E}_\ell^{(E,H)}(\vec{r},t) = \int \vec{E}_\ell^{(E,H)}(\vec{\kappa},\omega) \times \exp[i(\vec{\kappa}\vec{\rho} + k_{y\ell}y - \omega t)]dk_x dk_z d\frac{\omega}{v_0}, \tag{36}$$

where $k_{y\ell}$ are defined in Eq. (3). From the Maxwell Eqs. (4) and (5) we obtain the following expressions for the Fourier components of electric and magnetic fields of E and H types:

$$E_{y\ell}^{(E)}(\vec{\kappa},\omega) = -\frac{\kappa^2}{k_{y\ell}k_x}E_{x\ell}^{(E)}(\vec{\kappa},\omega), \tag{37}$$

$$E_{z\ell}^{(E)}(\vec{\kappa},\omega) = \frac{k_z}{k_x}E_{x\ell}^{(E)}(\vec{\kappa},\omega), \tag{38}$$

$$H_{x\ell}^{(E)}(\vec{\kappa},\omega) = \frac{\omega\varepsilon_\ell k_z}{ck_x k_{y\ell}}E_{x\ell}^{(E)}(\vec{\kappa},\omega), \tag{39}$$

$$H_{z\ell}^{(E)}(\vec{\kappa},\omega) = -\frac{\omega\varepsilon_\ell}{ck_{y\ell}}E_{x\ell}^{(E)}(\vec{\kappa},\omega), \tag{40}$$

$$E_{z\ell}^{(H)}(\vec{\kappa},\omega) = -\frac{k_x}{k_z}E_{x\ell}^{(H)}(\vec{\kappa},\omega), \tag{41}$$

$$H_{x\ell}^{(H)}(\vec{\kappa},\omega) = -\frac{ck_x k_{y\ell}}{\omega k_z}E_{x\ell}^{(H)}(\vec{\kappa},\omega), \tag{42}$$

$$H_{y\ell}^{(H)}(\vec{\kappa},\omega) = \frac{c\kappa^2}{\omega k_z}E_{x\ell}^{(H)}(\vec{\kappa},\omega), \tag{43}$$

$$H_{z\ell}^{(H)}(\vec{\kappa},\omega) = -\frac{ck_{y\ell}}{\omega}E_{x\ell}^{(H)}(\vec{\kappa},\omega). \tag{44}$$

The superposition of the electron fields in Eqs. (32) and (33), and transition radiation fields in Eqs. (37)–(44) must satisfy the boundary conditions which are analogous to Eqs. (8)–(10)

$$\{D_y\}_{y=0} = 4\pi e n^{2D}, \tag{45}$$

$$\{E_x\}_{y=0} = 0,\quad E_{z\ell}^e(0) + E_{z\ell}^{(E)}(0) + E_{z\ell}^{(H)}(0) = 0, \tag{46}$$

$$\{H_x\}_{y=0} = -\frac{4\pi}{c}j_z^{2D},\quad \{H_y\}_{y=0} = 0, \tag{47}$$

$$\{H_z\}_{y=0} = 0. \tag{48}$$

Here the braces denote the discontinuity of a corresponding field component. Upon substituting Eqs. (32), (33), and (37)–(44) into the boundary conditions (45)–(48) we obtain the following expressions for the Fourier components $E_{x\ell}^{(H)}(\vec{\kappa},\omega)$ and $E_{x\ell}^{(E)}(\vec{\kappa},\omega)$:

$$E_{x1}^{(H)}(\vec{\kappa},\omega) = E_{x2}^{(H)}(\vec{\kappa},\omega) = E_x^{(H)}(\vec{\kappa},\omega) = \frac{\omega^2 k_z^2 \Delta_1(\vec{\kappa},\omega)}{c^2\kappa^2\Delta_0(\vec{\kappa},\omega)}, \tag{49}$$

$$E_{x\ell}^{(E)}(\vec{\kappa},\omega) = \frac{k_x^2}{k_z^2}E_x^{(H)}(\vec{\kappa},\omega) - E_{x\ell}^e(\vec{\kappa},\omega), \tag{50}$$

where $k_y^e = \omega/v_0$

$$\Delta_0(\vec{\kappa},\omega) = \frac{1}{k_{y2}}\left(\frac{\omega^2}{c^2}\varepsilon_2 - k_z^2\right) - \frac{1}{k_{y1}}\left(\frac{\omega^2}{c^2}\varepsilon_1 - k_z^2\right). \tag{51}$$

$$\Delta_1(\vec{\kappa},\omega) = \varepsilon_2\left(\frac{1}{k_{y2}} - \frac{1}{k_y^e}\right)E_{x2}^e(\vec{\kappa},\omega) - \varepsilon_1\left(\frac{1}{k_{y1}} - \frac{1}{k_y^e}\right)E_{x1}^e(\vec{\kappa},\omega). \tag{52}$$

The rest of the radiation-field components can be expressed in terms of $E_{x\ell}^{(H)}(\vec{\kappa},\omega)$ and $E_{x\ell}^{(E)}(\vec{\kappa},\omega)$ using Eqs. (37)–(44). Besides, we can obtain the following expression for the surface current density $j_z^{2D}(\vec{\kappa},\omega)$:

$$j_z^{2D}(\vec{\kappa},\omega) = \frac{\omega\kappa^2}{4\pi k_x k_z}\left[\left(\frac{\varepsilon_2}{k_{y2}}E_{x2}^e(\vec{\kappa},\omega) - \frac{\varepsilon_1}{k_{y1}}E_{x1}^e(\vec{\kappa},\omega)\right) - \frac{k_x^2}{k_z^2}\left(\frac{\varepsilon_2}{k_{y2}} - \frac{\varepsilon_1}{k_{y1}}\right)E_x^{(H)}(\vec{\kappa},\omega)\right]. \tag{53}$$

From Eq. (51) it is evident that the value $\Delta_0(\vec{\kappa},\omega)$ in the denominators of Eqs. (49) and (50) is the dispersion relation of the SEMWs previously described. In order to derive the spatial and time dependencies of the SEMW fields in the explicit forms, we need to integrate expressions (37)–(44) with respect to k_x, k_z, and ω taking into account the poles of integrands in Eqs. (49) and (50).[13] As a matter of fact, it means that the electron traversing the infinite one-dimensional array of parallel thin metal wires placed in the interface plane of two different dielectrics can excite the SEMWs. Note that the SEMWs excitation is caused by that of the surface current [with the Fourier component in Eq. (53)] in metal wires.

IV. EXCITATION OF THE SEMWs BY THE BEAM-INSTABILITY EFFECT

Let an electron beam propagate in the half space $y<0$ in the positive Ox direction at a velocity $v_0 \ll c$. We suppose that the electron-beam width is much larger than the wavelength of the SEMW excited. Therefore, we will consider the electron beam as a semifinite one that occupies all half space $y<0$. As indicated in the previous section, we assume that $\sigma\to\infty$ and the radiation fields in both half spaces to be the superpositions of E—and H-type EMWs with components in Eqs. (34) and (35).

We consider that the spatial and time dependencies of E—and H-type EMW fields are harmonic ones, i.e., they are specified in the same way as the fields in Eqs. (1) and (2). In half space 2 the EMWs obey homogeneous Maxwell Eqs. (4)

and (5) at $\ell = 2$. In the half space 1 we describe the fields using the Maxwell equations concurrently with the linearized continuity and motion equations in small velocity- and electron-density-related perturbations $\vec{v}(\vec{r},t)$ and $n(\vec{r},t)$. The corresponding set of coupled equations are

$$\nabla \times \vec{H}_1 = \frac{1}{c}\frac{\partial \vec{E}_1}{\partial t} + \frac{4\pi}{c}\vec{j}_B(\vec{r},t), \quad \nabla \times \vec{E}_1 = -\frac{1}{c}\frac{\partial \vec{H}_1}{\partial t}, \tag{54}$$

$$\mathrm{div}\, \vec{E}_1 = 4\pi n(\vec{r},t), \quad \mathrm{div}\, \vec{H}_1 = 0, \tag{55}$$

$$\frac{\partial n(\vec{r},t)}{\partial t} + n_0\, \mathrm{div}\, \vec{v}(\vec{r},t) + \mathrm{div}[n(\vec{r},t)\vec{v}_0] = 0, \tag{56}$$

$$\frac{\partial \vec{v}(\vec{r},t)}{\partial t} + \vec{v}_0 \frac{\partial}{\partial \vec{r}}\vec{v}(\vec{r},t) = \frac{e}{m_0}\left(\vec{E}_1 + \frac{1}{c}[\vec{v}_0,\vec{H}_1]\right), \tag{57}$$

where $\vec{j}_B$ is the electron-beam current-density perturbation

$$\vec{j}_B(\vec{r},t) = en_0\vec{v}(\vec{r},t) + en(\vec{r},t)\vec{v}_0 \tag{58}$$

n_0 is the mean electron-beam density, $\vec{v}_0 = (v_0, 0, 0)$. Note that Eqs. (54)–(58) is the starting point for the general analysis of the beam-instability effects. Solving the equation set in Eqs. (54)–(58) we obtain the following expressions for the components of the electron-beam current-density perturbations:

$$j_{xB} = \frac{ie^2 n_0 \omega E_{x1}}{m_0(\omega - k_x v_0)^2} + \frac{ie^2 n_0 k_z v_0}{m_0(\omega - k_x v_0)^2}\left(E_{z1} + \frac{v_0}{c}H_{y1}\right) + \frac{e^2 v_0}{m_0(\omega - k_x v_0)^2}\frac{\partial}{\partial y}\left[n_0\left(E_{y1} - \frac{v_0}{c}H_{z1}\right)\right], \tag{59}$$

$$j_{yB} = \frac{ie^2 n_0}{m_0(\omega - k_x v_0)}\left(E_{y1} - \frac{v_0}{c}H_{z1}\right), \tag{60}$$

$$j_{zB} = \frac{ie^2 n_0}{m_0(\omega - k_x v_0)}\left(E_{z1} + \frac{v_0}{c}H_{y1}\right). \tag{61}$$

Substituting Eqs. (59)–(61) into the first curl equation in Eq. (54) and integrating with respect to y in the near vicinity of the point $y=0$, we obtain the expression for the discontinuity of the H_z filed component crossing the interface plane (see Ref. 14)

$$\{H_z\}_{y=0} = \frac{4\pi}{c}\lim_{\eta\to 0}\int_{-\eta}^{\eta} j_{xB}\,dy = -\frac{\omega_B^2 v_0}{c(\omega - k_x v_0)^2}\left[E_{y1}(0) - \frac{v_0}{c}H_{z1}(0)\right], \tag{62}$$

where $\omega_B = \sqrt{4\pi e^2 n_0/m_0}$ is the electron-beam plasma frequency. The simultaneous solution of boundary conditions (46), (47), and (62) yields the dispersion relation for the coupled SEMWs

$$\Delta_0(\vec{\kappa},\omega)(\omega - k_x v_0)^2 = \frac{\omega_B^2 v_0}{c k_{y2}}\left[\frac{\omega}{c}k_x - \frac{v_0}{c}\left(\frac{\omega^2}{c^2} - k_z^2\right)\right], \tag{63}$$

where $\Delta_0(\vec{\kappa},\omega)$ is defined by Eq. (51)

$$k_{y1} = -i\sqrt{\kappa^2 - \frac{\omega^2}{c^2}}, \quad k_{y2} = i\sqrt{\kappa^2 - \frac{\omega^2}{c^2}\varepsilon_2}. \tag{64}$$

It should be stressed that relation (63) is valid for a weak electron-beam density where the following conditions are met:[15]

$$\omega_B \ll \delta\omega \ll \omega, \tag{65}$$

where $\delta\omega$ is the instability increment for the coupled SEMWs. To derive the instability increment it is necessary to represent the wave frequency ω as

$$\omega = \omega_R + \delta\omega, \tag{66}$$

where $|\delta\omega| \ll \omega_R$ and ω_R is the resonance frequency at which the following conditions hold true:

$$\omega_R = k_x v_0, \quad \Delta_0(\vec{\kappa},\omega_R) = 0. \tag{67}$$

Substitution of Eq. (67) into Eq. (63) and the subsequent expansion in series of $\delta\omega$ leads to the cubic equation in $\delta\omega$ with real coefficients if the absorption is neglected. Another words, we obtain one real and two complex conjugate roots, and one of the complex roots corresponds to the beam-induced instability with the following increment:

$$\delta\omega = \frac{\sqrt{3}}{2}\left[\frac{\omega_B^2 v_0 \kappa^4}{\varepsilon_2 k_x (k_x^2 + \kappa^2)}\right]^{1/3}, \tag{68}$$

where $\varepsilon_2 v_0^2/c^2 \ll 1$. For the SEMWs propagating almost perpendicular to the wires expression (68) can be simplified as

$$\delta\omega \approx \frac{\sqrt{3}}{2}\left(\frac{\omega_B^2 k_x v_0}{2\varepsilon_2}\right)^{1/3}. \tag{69}$$

We demonstrate that it is exactly the SEMWs propagating almost perpendicular to the wires are excited by the beam. Indeed, from Eqs. (15) and (67) we can get the resonance angle ϑ_R for excited waves by equating frequency $\omega_{(-)}$ to the ω_R. As a consequence, we have

$$\vartheta_R = \frac{\pi}{2} - \frac{v_0}{c}\sqrt{\frac{1+\varepsilon_2}{2}} + O\left(\frac{v_0^2}{c^2}\right). \tag{70}$$

It should noted that, as seen from Eq. (70), the electron beam propagating perpendicular to the wires tend to excite the well-bounded SEMWs with dispersion relation in Eq. (17).

Now we present the results of the numerical calculations of $\delta\omega$ and ϑ_R for the THz-frequency region. For the widely used parameters,[14] $k_x = 5\times 10^4$ m^{-1}, $v_0 = 0.1c$ ($\omega_R \simeq 1.5 \times 10^{12}$ s^{-1}), and $n_0 = 10^{15}$ m^{-3} ($\omega_B \simeq 1.8\times 10^9$ s^{-1}), $\varepsilon_2 = 2$, we find that $\delta\omega \simeq 9.2\times 10^9$ s^{-1} and $\vartheta_R \simeq 85°$.

V. CONCLUSION

In this paper we have theoretically analyzed the possibility of SEMWs propagation along a flat interface between the two different dielectric media in the case where an one-dimensional array of thin metal wires is placed at the interface. Our supposition was that the distance between the neighboring wires is far less than the wavelength and the surface wave frequencies are far lower than the plasma frequency of a metal. The existence conditions, dispersion relation, energy flux, and angle distribution of the SEMWs frequency have been derived. Specifically, the wave propagation is shown to be excluded when the array of wires is sandwiched between two identical dielectrics. In addition, it has been found that the SEMWs are well bounded and the highly delayed ones in both dielectric media mainly in propagation directions almost perpendicular to the wires. We have shown that it is possible to excite the SEMWs by means of the transition-radiation and beam-instability effects. Besides, one can expect the SEMWs to propagate at long distances due to negligible energy losses in surrounding dielectric media. These facts along with the simple interface design open up the possibility of practical applications of the SEMWs.

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